Examining the development of mathematics education in China over the past three decades: A focused survey

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Outline

1. Introduction and Background

2. Research Methods

3. Findings and Results

4. Concluding remarks: Possible directions and trends

1. Introduction and Background

During this period, mathematics education has also undergone some highly noteworthy reforms, changes and developments in research, curriculum and classroom practices.
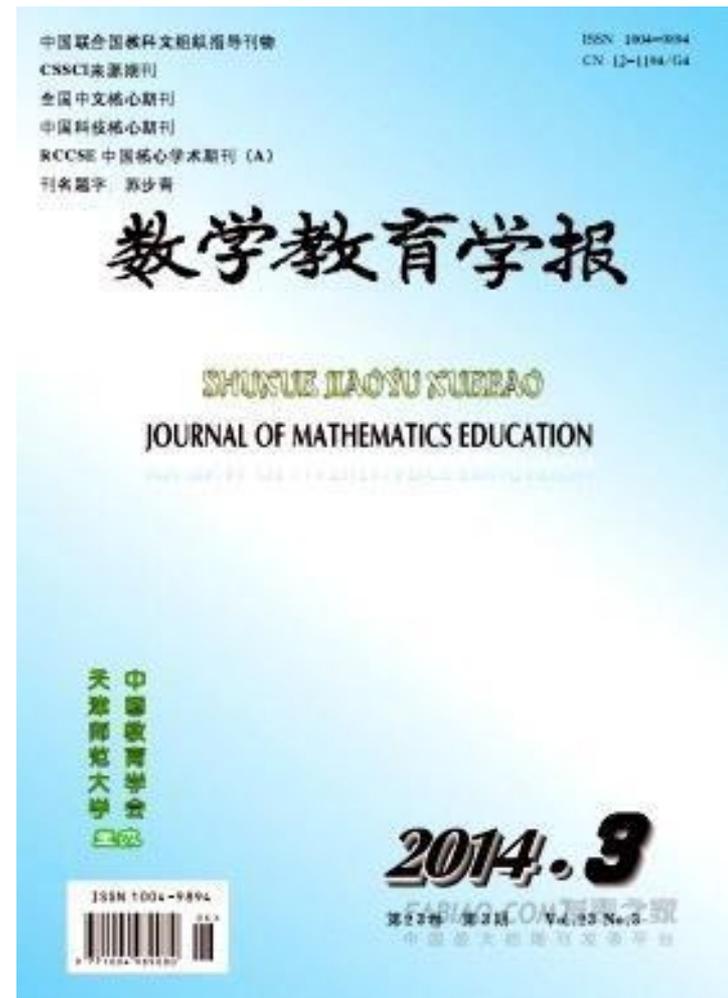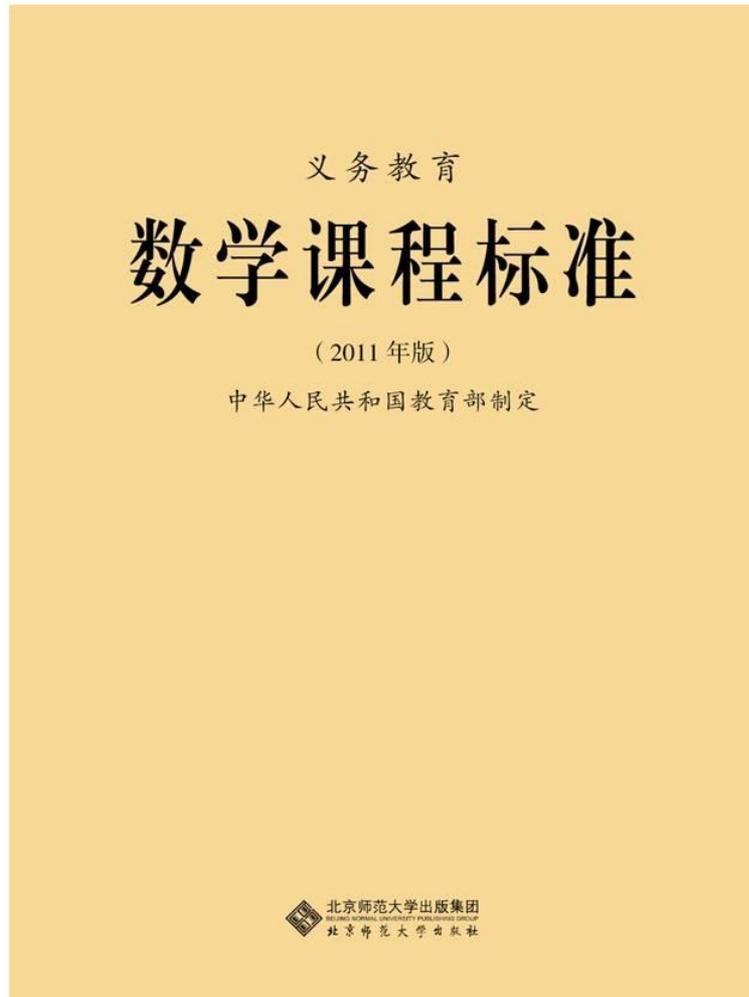
In this speech, I will present how the research and practice of mathematics education in China has evolved since the early 1990s.

Research Assistants for this study:

Luo, Jietong

Xie, Sicheng

2. Research Methods

- We conducted a **survey study** of articles published over the last three decades.

- Most of our attention was paid to two most frequently referred journals in mathematics education in China:

    Shuxue Jiaoyu Xubao (*Journal of Mathematics Education*) ;

    Shuxue Tongbao (*Journal of Mathematics*)

- Other sources also received some attention.

Most frequently cited journals in mathematics education

Category

Journal of Mathematics Education

Journal of Mathematics

表 1 数学教育类期刊一览表

| 序号	刊名	主办单位	类别	创刊时间	被引次数
1	《数学教育学报》	天津师范大学;中国教育学会	中文核心;CSSCI	1992	58798
2	《数学通报》	中国数学会;北京师范大学	中文核心	1936	32716
3	《数学学习与研究》	吉林省数学会；东北师范大学		1983	15221
4	《中学数学教学参考》	陕西师范大学		1972	14539
5	《中学数学》	湖北大学		1979	11443
6	《数学通讯》	华中师范大学;湖北省数学学会;武汉数学学会		1933	10897
7	《数学教学》	华东师范大学		1955	7795
8	《数学教学通讯》	重庆市数学学会;西南大学		1979	7183
9	《中国数学教育》	中国教育学会中学数学教学专业委员会；辽宁北方教育报刊出版有限公司		2003	6416
10	《中学数学月刊》	苏州大学;江苏省数学学会		1978	6414

Note:
1. Data source: cnki.net
2. Number of citations until May 9th, 2019.
3. The column "category" indicates the importance of the journals.

Some other sources:

华人数学教育研究系列 （英文版）

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Perspectives from Insiders
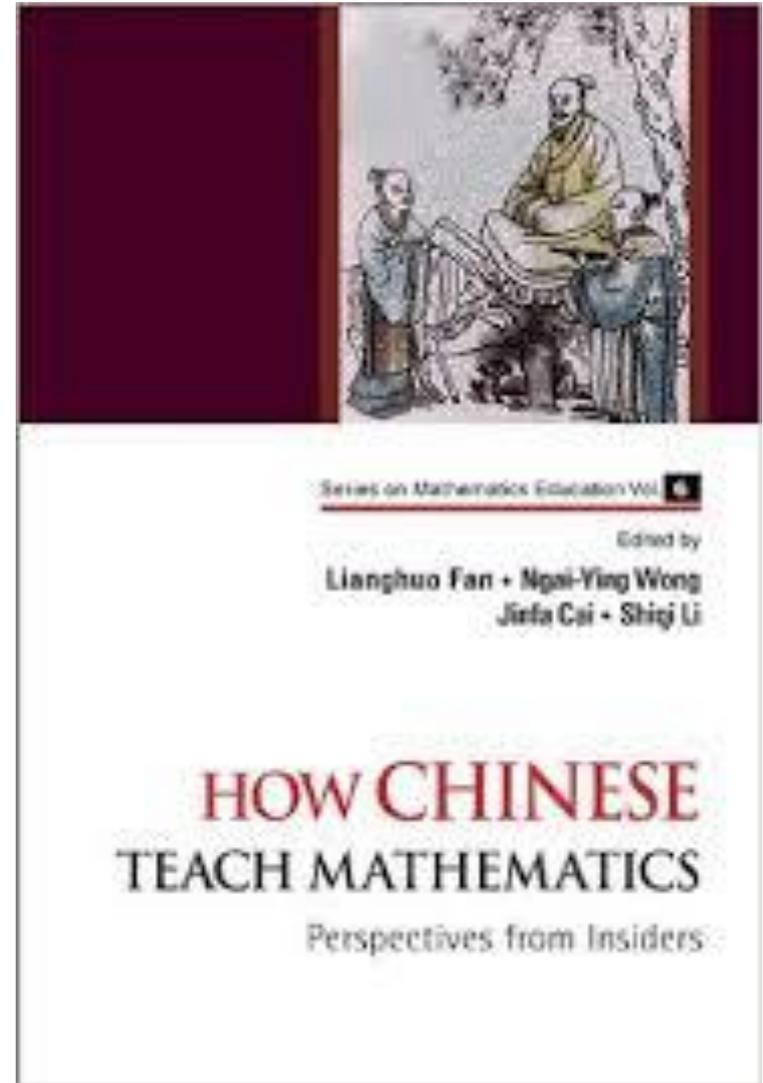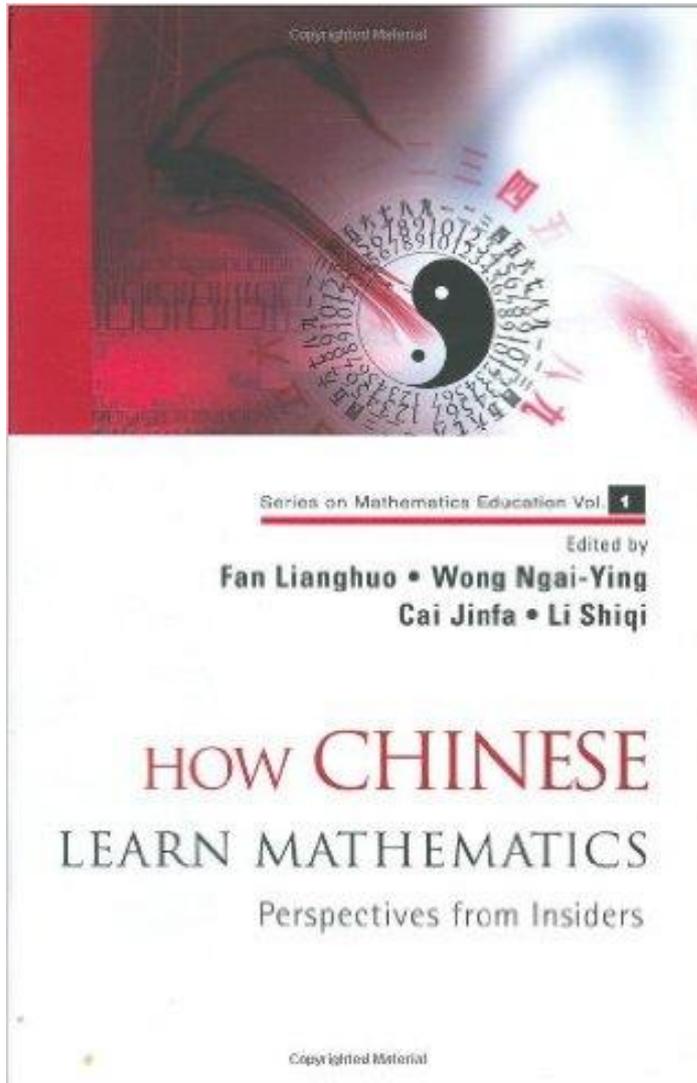
华人数学教育研究系列 （中文版）
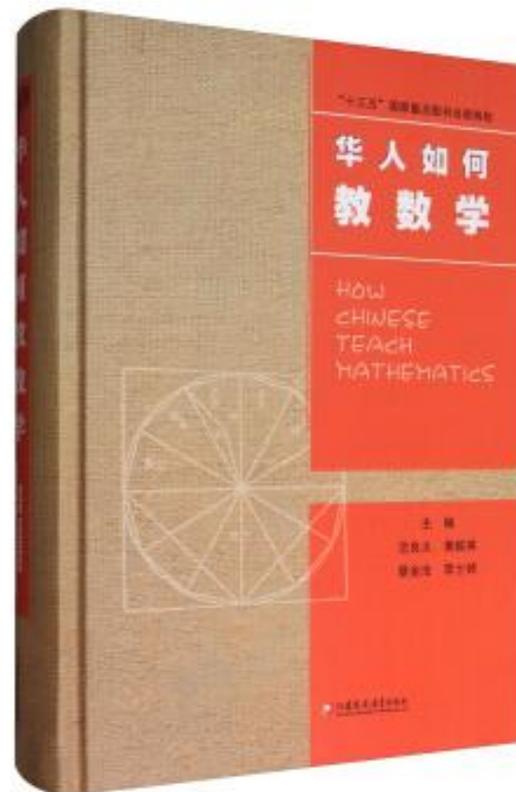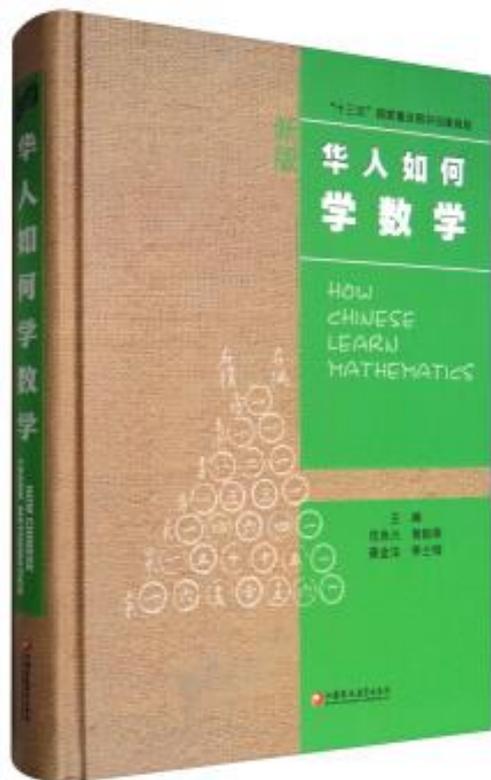
Focuses of Survey

- ✓ Topics/Issues

- ✓ Methods

Journal of Mathematics Education (JME)

- Year of launch: 1992.

- Frequency of publication:
  - ✓ 1992: 1 issue;
  - ✓ 1993-1994: biannually
  - ✓ 1995-2007: quarterly;
  - ✓ 2008-: bimonthly.

- Each issue consists of 20-30 articles, and sometimes includes some news and announcements.

Classifications for Coding

- **Research methods：**

    Empirical vs non-empirical

- **Topics/Issues:**

    (1) mathematics teaching and learning,

    (2) mathematics curriculum and textbook research,

    (3) mathematics teacher education and professional development,

    (4) mathematical culture and history in mathematics education, and

    (5) Others.

- **Education stages：**

    Kindergarten, primary, secondary, university, etc.

Classification 1 (Research methods)

Empirical vs non-empirical

Classification 2 (Topics/Issues)

Area	Description
1. Mathematics teaching and learning	<u>Including articles about: (1a) Mathematics classroom teaching , (1b) Mathematics classroom learning, (1c) ICT in mathematics education, (1d) Psychology of mathematics teaching and learning, and (1e) Mathematics education assessment, measurement and evaluation.</u>
2. Mathematics curriculum and textbook research	Studies about (2a) Curriculum and (2b) Textbook. There are also some topics that we highlighted, and their codes will be seen in later pages.
3. Mathematics teacher education and professional development	Studies about (3a) Mathematics pre-service teacher education and (3b) Mathematics in-service teacher education and professional development. We also identified articles about (3c) Knowledge of mathematics teachers.
4. Mathematical culture and history in mathematics education	Literature about (4a) Mathematics culture in mathematics education and (4b) Mathematics history in mathematics education. There is also a Chinese characteristic subarea called (4c) Mathematics education of students of ethnic minorities.
5. Others	Including (5a) issues, methods and philosophy of mathematics education (detailed code of topics see later pages), (5b) Mathematics thinking and methodology, (5c) Study of Examinations of Gaokao and Zhongkao, (5d) STEM and (5e) Miscellaneous (detailed code of topics see later pages).

Sub-classification 1:
Mathematics teaching and learning

Area 1: Mathematics teaching and learning	
(1a) Mathematics classroom teaching	Regarding how teachers teach mathematics in classroom environment, problem solving, problem posing, etc. We also highlighted a Chinese characteristic topic: **teaching with variations** (where we highlighted the works of Prof Gu Lingyuan, Honorary Professor of ECNU).
(1b) Mathematics classroom learning	Regarding how students learn mathematics in the classroom environment
(1c) ICT in mathematics education	Regarding the use of Information and Communication Technology in mathematics education
(1d) Psychology of mathematics teaching and learning	Regarding the psychological aspects of mathematics teaching and learning
(1e) Mathematics education assessment, measurement and evaluation	Regarding the assessment, measurement and evaluation of students' study outcome and teachers' teaching effect by conducting classroom activities.

Sub-classification 2: Mathematics curriculum and textbook

Area 2: Mathematics curriculum and textbook	
(2a) Curriculum	Concerning about all the curricula excluding pre-service mathematics teacher curricula. One main topic throughout this time period is about **curriculum reforms**. There is also a topic that is worth being classified, which is **(2a1) International comparative study of mathematics curriculum**.
(2b) Textbook research	All textbook research, where the following topic worth being classified: **(2b1) International comparative study of mathematics textbook**.

Sub-classification 3:
Mathematics teacher education and professional development

Area 3: Mathematics teacher education and professional development	
(3a) Mathematics pre-service teacher education	About how to develop pre-service teachers' skills in normal colleges, including curriculum setup and teaching methods in normal college courses, developing pre-service teachers' pedagogy skills and qualities, internship arrangement, etc.
(3b) Mathematics in-service teacher education and professional development	About how to improve in-service teachers' qualities and pedagogy skills, and about their professional development.
(3c) Knowledge of mathematics teachers	Including PCK, MKT, MPCK, etc.

Sub-classification 4:
Mathematical culture and history in mathematics education

Area 4: Mathematics culture and history in mathematics education	
(4a) Mathematical culture in mathematics education	Regarding cultural factors in mathematics education. An article is classified into this subarea only if the word "culture" is mentioned in its title, abstract or key words.
(4b) Mathematics history in mathematics education	Regarding how to include mathematics history knowledge into mathematics education. We highlighted a very important topic **History and Pedagogy of Mathematics (HPM)** in this subarea, where we highlighted the works of Prof Wang Xiaoqin from ECNU.
(4c) Mathematics education of students of ethnic minorities	A Chinese characteristic theme. Studies specifically focusing on how minority races in China study mathematics, how their unique cultural factors affect their mathematics education, and how we can cultivate their cognition to their own culture in mathematics education through some mathematics cultural factors occurred in their culture.

Sub-classification 5: Others

Area 5: Others	
(5a) Issues, methods and philosophy in mathematics education	Including the following topics: **(5a1) Philosophy of mathematics education**, **(5a2) Mathematical aesthetic**, **(5a3) Fundamental of mathematics education**, and **(5a4) Meta-study of mathematics education**.
(5b) Mathematics thinking and methodology	A Chinese characteristic topic. Mainly non-empirical articles discussing how to develop mathematics thinking (e.g. Thought of transformation, Ability of posing a conjecture, etc.) and methods in problem solving.
(5c) Study of Examinations of Gaokao and Zhongkao	A Chinese characteristic topic. Studies focusing on the trend of Gaokao and Zhongkao reforms, year-by-year problem characteristics, paper setting, etc.
(5d) STEM	Studies about Science, Technology, Engineering and Mathematics (STEM).
(5e) Miscellaneous	Including **Academic Conference summary (5e2)**, which is another Chinese characteristic topic, and broadly all the other studies or articles in mathematics education. Some **News (5e1)** are recorded and when calculating final results, we deleted them.

Classification 3 (Stages of education)

Stage		Description
A. Preschools		Mostly kindergarten
Compulsory education	B. Primary school	6 years (Shanghai: 5 years)
	C. Junior high school	3 years (Shanghai: 4 years)
	D. Overall	
E. Junior and senior high school		6 years (Shanghai: 7 years), the so-called "secondary education stage"
High school education	F. Senior high school	3 years, admission by Zhongkao, mostly studying basic knowledge for all subjects
	G. Vocational high school	3 years, mostly training certain technical skills, knowledge is secondary concern
Tertiary education	H. Normal university (mathematics)	4 years, concerning students major in mathematics in normal universities (i.e. pre-service teachers)
	I. Non-normal university/ Normal university (non-maths)	3-5 years, admission by Gaokao, concerning students that do not major in mathematics in normal universities
J. Others		Including adults education, general studies, etc.

The overall framework of classification

3. Findings and Results

Research results

- **Research methods**：

    Empirical vs non-empirical

- **Topics/Issues:**

    (1) mathematics teaching and learning,

    (2) mathematics curriculum and textbook research,

    (3) mathematics teacher education and professional development,

    (4) mathematical culture and history in mathematics education, and

    (5) Others.

- **Education stages**：

    Kindergarten, primary, secondary, university, etc.

Results (Classification 1)

Distribution of research types (1990s and 2010s)
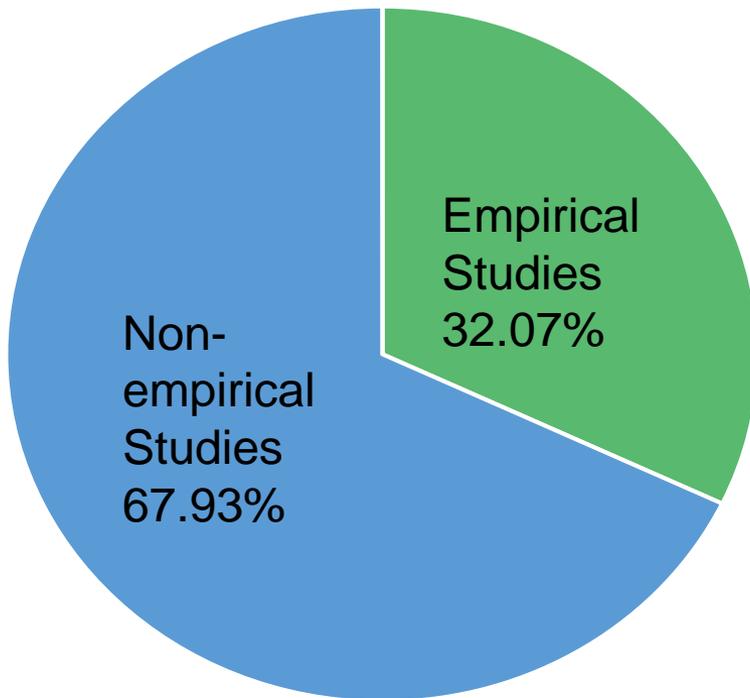
Fig. 1 Distribution of research types in the 1990s (n=658)
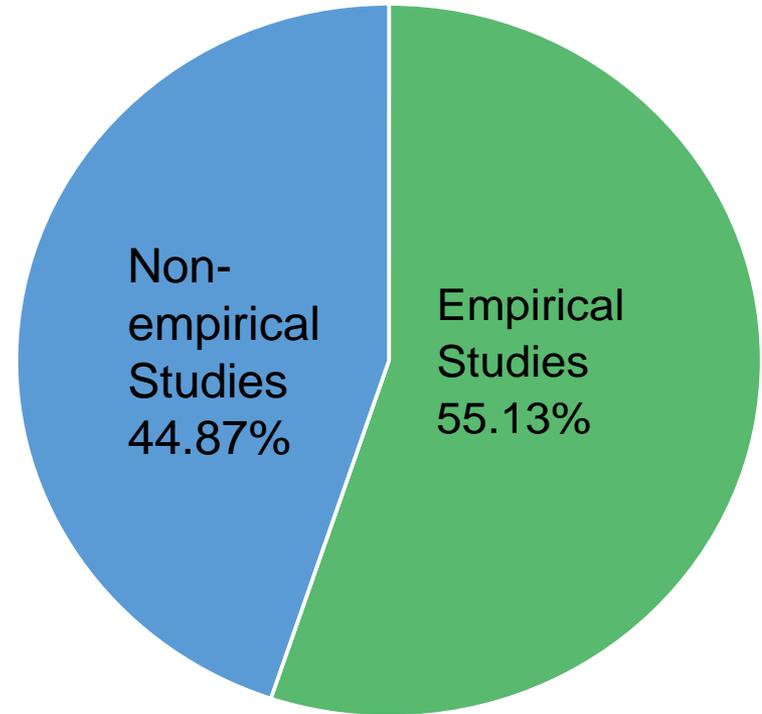
Fig. 2 Distribution of research types in the 2010s (n=1315)

Note: Inter-rater consistency: 90%

Results (Classification 2)

3.1. Mathematics classroom teaching and learning

1. Teaching with variation（变式教学）

- Intention: to illustrate the essential features by demonstrating different forms of visual materials and instances or highlight the essence of a concept by varying the nonessential features.

- Aim: understanding the essence of object and forming a scientific concept by putting away nonessential features.

(from: Gu,1999)

Conceptual variation:
## Understanding concepts from multiple perspectives
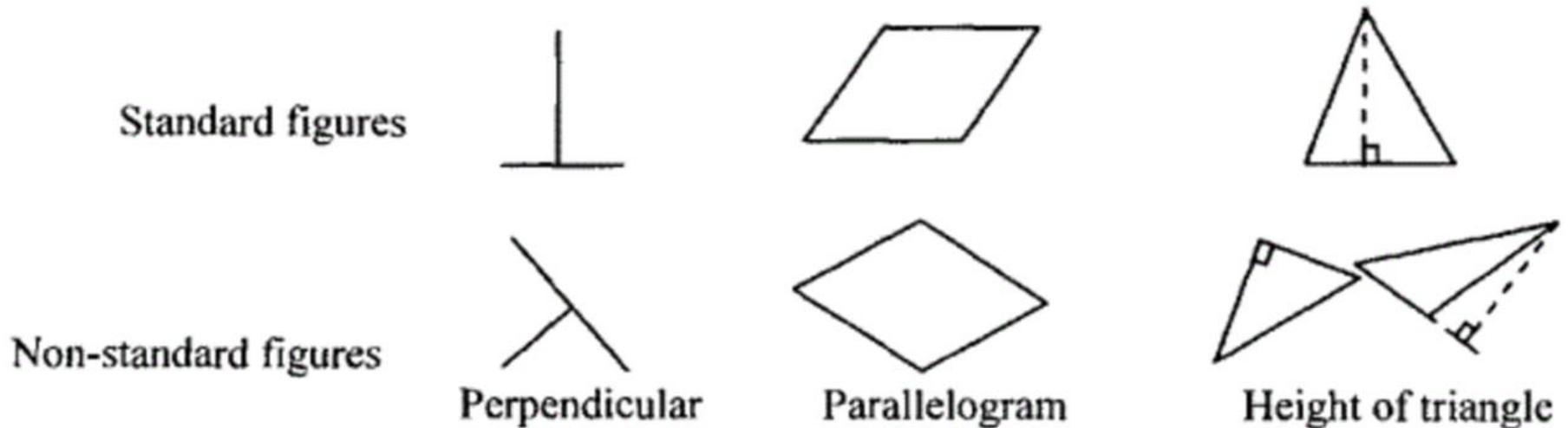
Fig. 3 Geometrical standard figure and non-standard figure variation (L. Gu et al., 2004)

Procedural variation:
Progressively unfolding mathematics activities

Scaffolding 1: Representing the unknown by concrete things.

For example, "James pays 2 $D for buying three rubbers and the seller gives him 2 coins in change (1/10 $D). How much is for each rubber?" The following expression can present the question visually.

$$2D - \boxed{\diamond}\,\boxed{\diamond}\,\boxed{\diamond} = 2C \qquad \text{or} \qquad 2D - 3\boxed{\diamond} = 2C \qquad (1)$$

Scaffolding 2: Symbolizing the unknowns.

$$2D - 3x = 2C \qquad (2)$$

or

$$20 - 3x = 2 \qquad (3)$$

Scaffolding 3: Replacing unknown x with symbolic "□".

$$20 - 3\square = 2 \qquad (4)$$

(L. Gu et al., 2004)

# 一课一练 数学 （英国版和上海英文版）
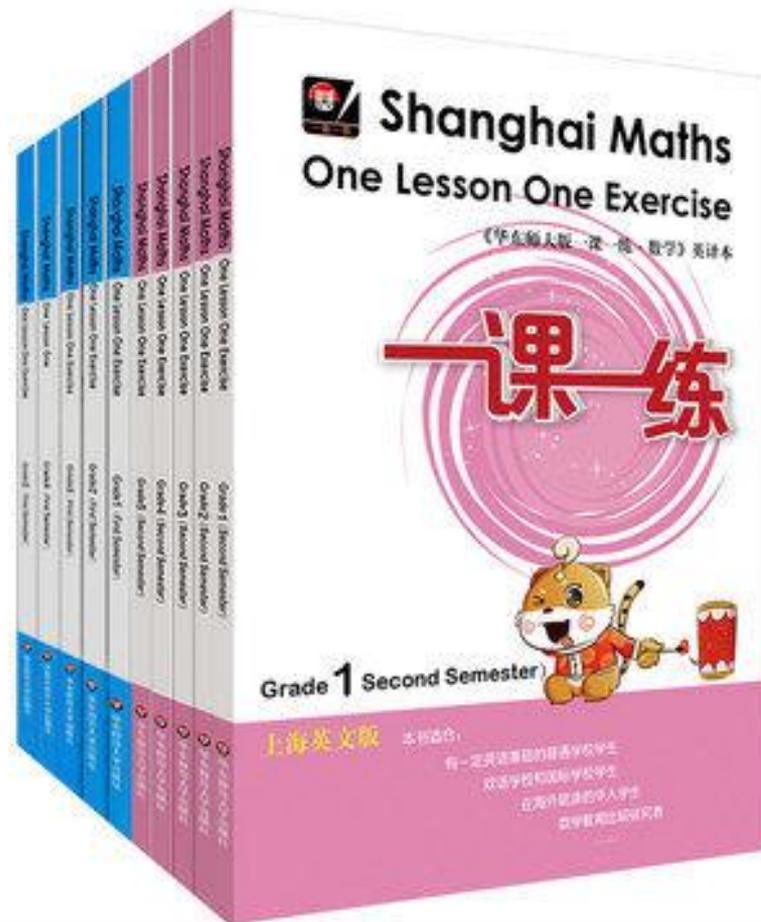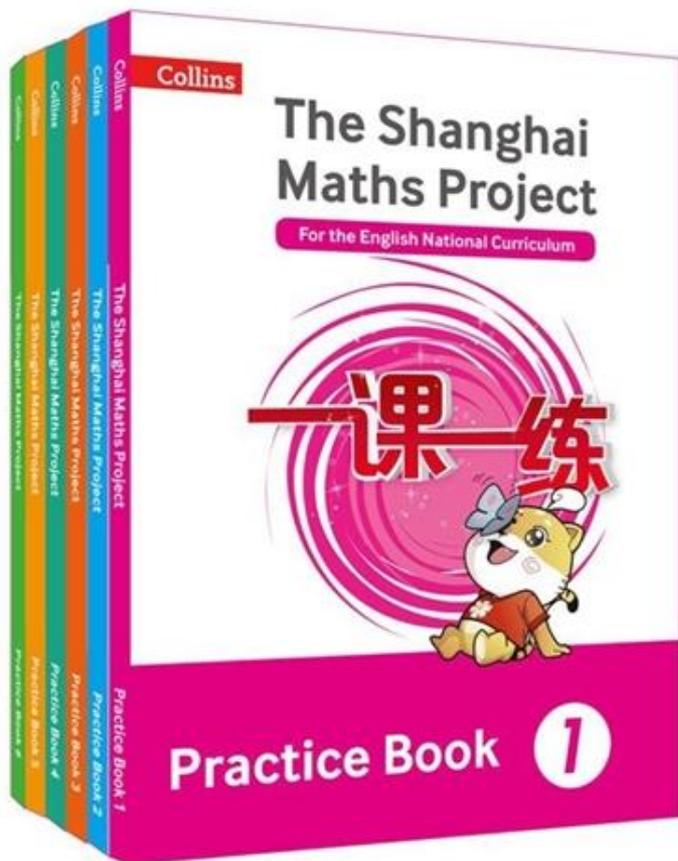
2. Open-ended questions
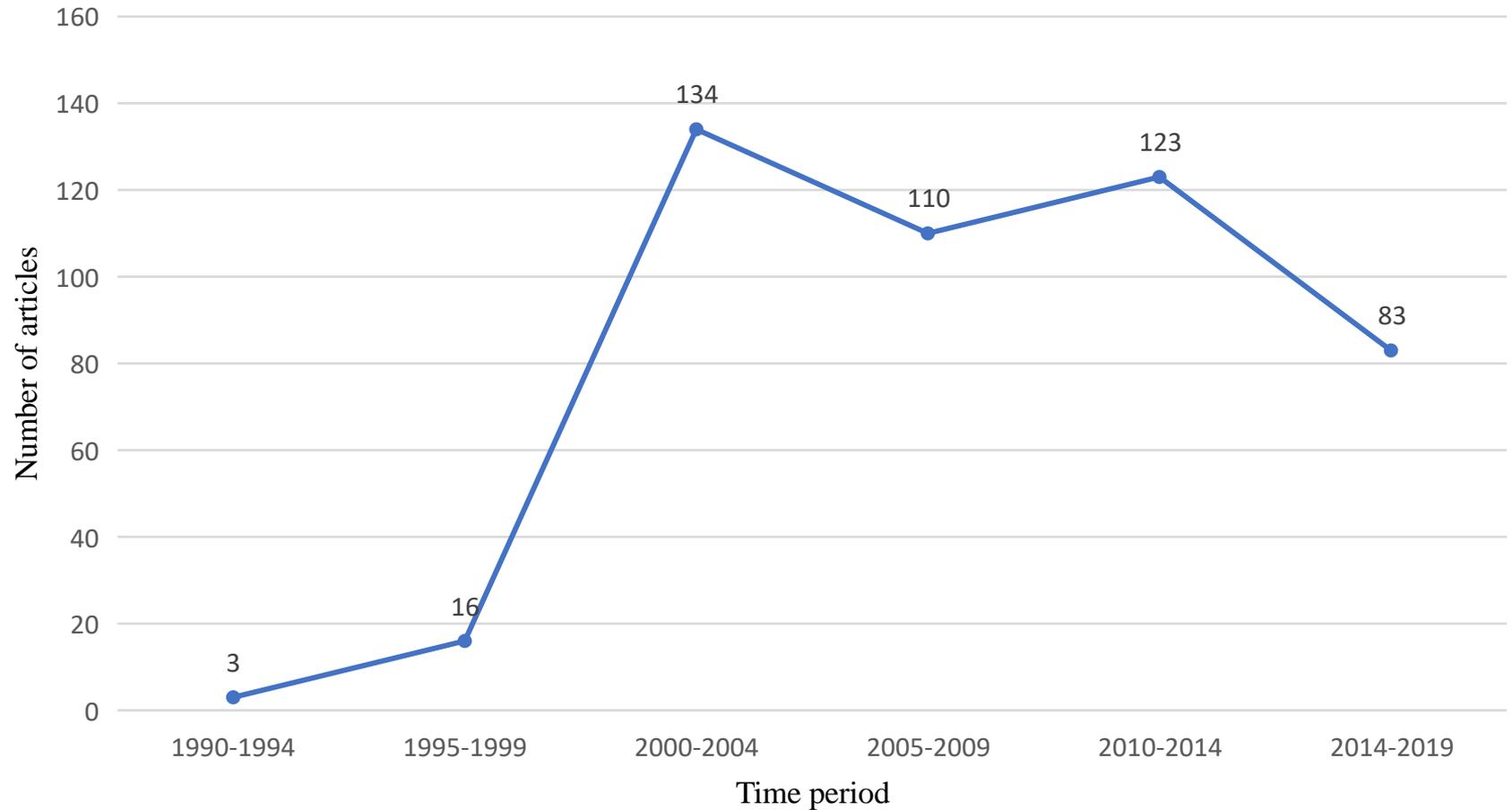
Fig.4 Distribution of articles[1] on open-ended questions between 1990 and 2019 (n=469)

1. Data source: cnki.net

Open-ended questions

- Learned from Japan.

- Studies on this field did not attract Chinese researchers' interest until 1993, when some teaching experiments of open-ended questions have been conducted (Z. Dai 1993).

- After this year, a series of studies on open-ended questions, including how to design open-ended questions, how to solve open-ended questions, the function of open-ended questions and open-ended questions in Gaokao, have come out (e.g., W. Chen 1996, Q. Hu 1998, K. Wang 1999, C. Qian, 1999).

Open-ended questions

- In 2000, MOE suggested that there could be more open-ended questions in Zhongkao.
  http://www.gov.cn/gongbao/content/2000/content_60151.htm

- With this document, more articles on open-ended problems in Zhongkao, also there were more studies exploring open-ended questions in mathematics classroom teaching at different stages (C. Xia 2003, D. Zhang & Z. Dai 2005).

- There were fewer studies focusing on open-ended questions published in JME in recent years. However, in other practical journals in mathematics education, it is still a hot topic, though showing some signals of downtrend.

3.2. Curriculum and Textbook research

Curriculum

Mathematics curriculum reform

- Compulsory Education
- Senior High Schools

2001: *Mathematics Curriculum Standards of Compulsory Education (Experimental Manuscript)*

Two Basics（双基）

2002: *General High School Mathematics Curriculum Standards (Experimental Manuscript)*

2012: *Mathematics Curriculum Standards of Compulsory Education (Version 2011)*

Four Basics（四基）

Core Literacy（六大核心素养）

2018: *General High School Mathematics Curriculum Standards (Version 2017)*

➢ Requirements

• **Two Basics（双基）**: basic knowledge (基础知识) and basic skills (基本技能)

• **Four Basics（四基）**: basic knowledge (基础知识), basic skills (基本技能), basic method in mathematics thinking (基本数学思想方法), and basic experience in mathematics activities (基本活动经验)

Basic experience in mathematics activities

Basic method in mathematics thinking

Basic skills

Basic knowledge
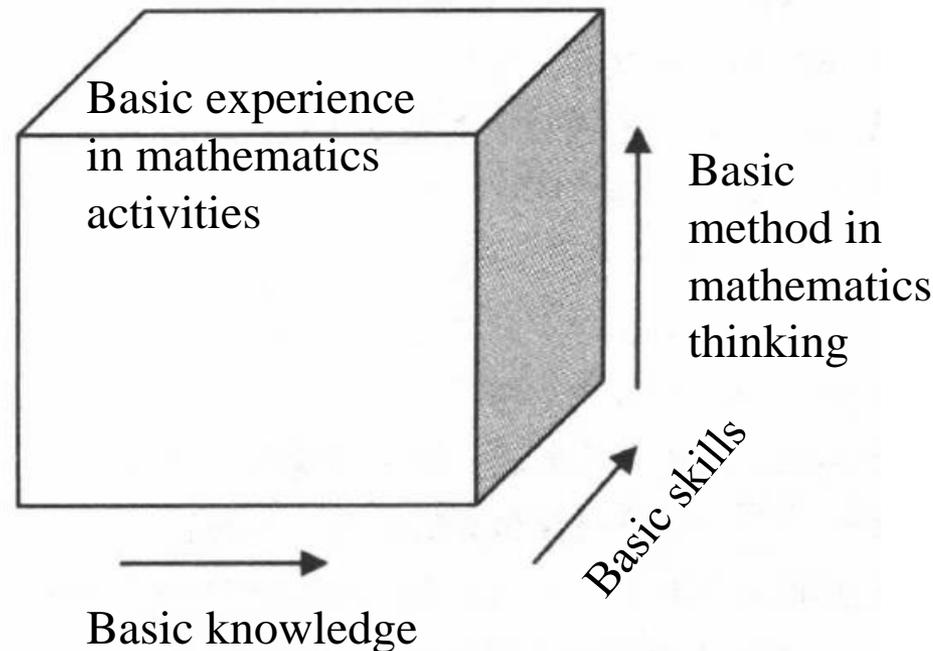
Fig. 5 A model of four basics (D. Zhang & Z. Zheng, 2011)

Core Literacy（六大核心素养）

- Mathematical abstraction (数学抽象)
- Logical reasoning (逻辑推理)
- Mathematical modeling (数学建模)
- Mathematical operation (数学运算)
- Intuitive imagination (直观想象)
- Data analysis (数据分析)

Textbook research

Table 1 Distribution of articles about textbook research in JME

Period	Topic	A	B	C	D	E	F	G	I	J	Subtotal	Total
1990-1994	2b	0	0	0	0	0	0	0	0	1	1	1
	2b1	0	0	0	0	0	0	0	0	0	0	
1995-1999	2b	0	0	5	1	5	2	0	1	2	16	19
	2b1	0	0	1	0	0	0	0	0	2	3	
2000-2004	2b	0	5	2	1	2	0	0	1	1	12	12
	2b1	0	0	0	0	0	0	0	0	0	0	
2005-2009	2b	0	1	5	4	0	4	0	0	2	16	20
	2b1	0	0	3	0	0	0	0	0	1	4	
2010-2014	2b	0	8	6	0	1	12	0	4	1	32	49
	2b1	0	4	5	0	1	7	0	0	0	17	
2015-2019	2b	0	5	7	1	1	3	0	0	6	23	44
	2b1	0	6	8	0	0	6	0	1	0	21	
Subtotal		0	29	42	7	10	34	0	7	16	144	144

Notes:
1. A=Preschool, B=Primary school, C=Junior high school, D=Senior high school, E=Junior and senior high school, F=Senior high school, G=Vocational high school, H=Normal university (mathematics), I=Non-normal university/Normal university (non-mathematics), J=Others, 2b=Textbook research (excluding international comparative study on textbooks), 2b1=International comparative study on textbooks.
2. Excluded research about textbooks for mathematics students in normal universities (i.e. pre-service teachers).

Textbook research
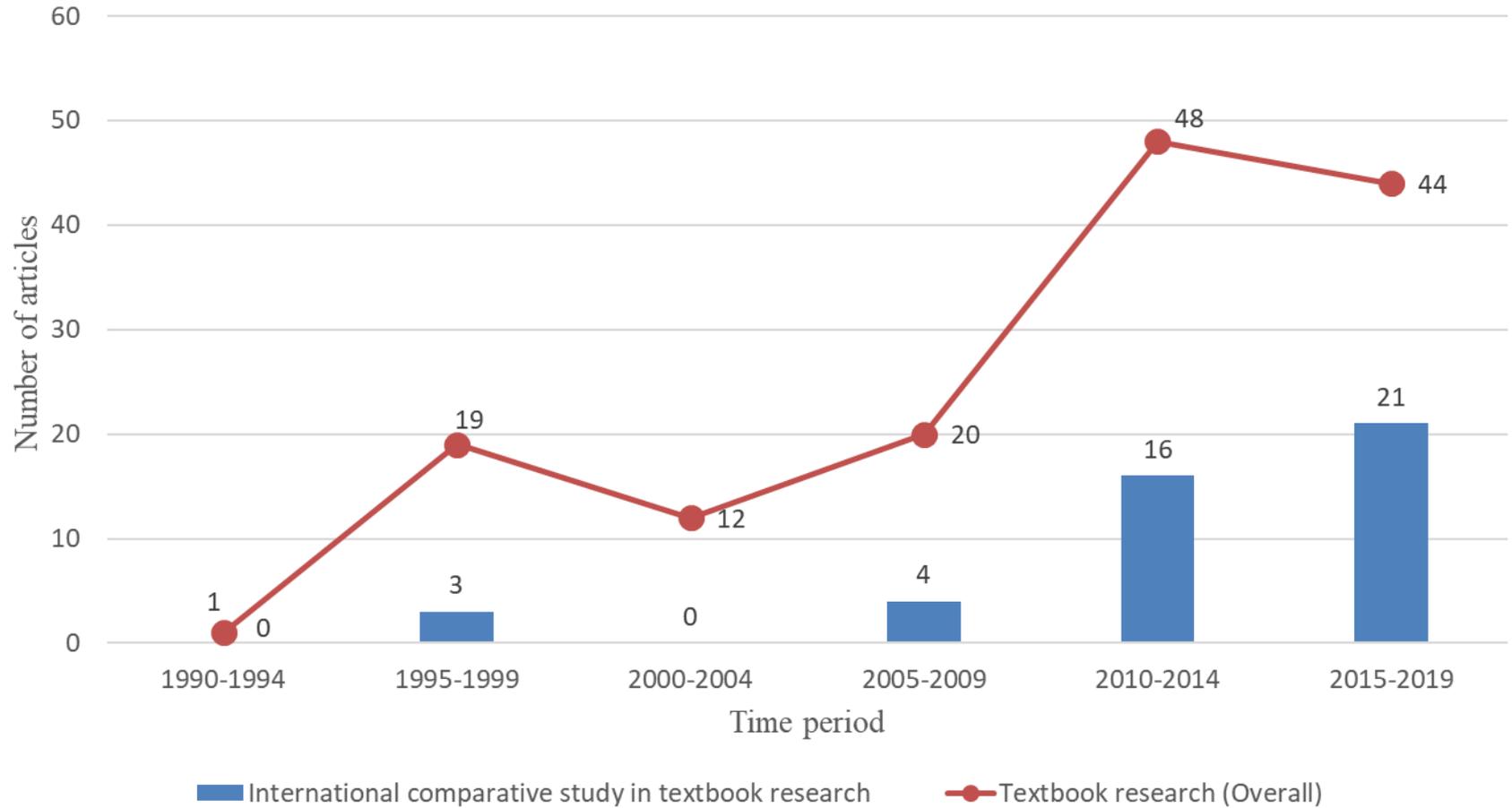
Fig. 7 Overall trend of development in textbook research (based on JME)

Textbook research

- In 1994, JME published the first article about textbook research (P. Yu, 1994). Since then, this topic has gradually become an important topic.

- In the 1990s, most studies in this field focused on how to design some specific topics in the textbooks. Later on, in accordance with curriculum reforms, many researchers started to pay attention to textbook comparison, especially rigid international textbook comparative study since 2005 (K. Li, 2005).

- Most studies were about textbooks in high schools (junior and senior high). Research about textbooks in primary schools has become more popular in this decade. But still, there is no study about pre-school and vocational high school textbooks.

- Most of studies on this field only looked into the text in the final published version of textbooks, whilst few studies are more directly targeted at the issues about textbook development (L. Fan & L. Wu, 2015).

3.3 Mathematics teacher education and professional development

- In the 1990s, most of the studies in this area were about pre-service mathematics teacher education, including how to set up curricula in the normal universities.

- After 2000, pre-service and in-service mathematics teacher education have both played important roles in this area. Meanwhile, the knowledge of mathematics teachers, such as PCK, MKT, MPCK, TPACK, etc., was emphasized by researchers in the recent 15 years.

Knowledge of teachers

**Table 2 Sources of literature surveyed in the study "how teachers develop their professional knowledge"**

	Before 2005	**2005-2009**	**2010-2014**	**2014-2018**
Publications[1]	1	7	13	3

1. Publications until Dec 31st, 2018.

- The research results have been increasing over the past 15 years, indicating that the survey of teachers' knowledge sources has received more and more attention from researchers.

- For the content of teacher knowledge, there are both an overall study of teacher knowledge and a specific field or branch that focuses on teacher knowledge.

- For the survey samples, both in-service teachers and pre-service teachers were included. The samples range from dozens to more than 1000 teachers.

- For the research methods, questionnaire surveys are more common, while some other studies use a variety of methods such as questionnaires, interviews, and classroom observations.

(D. Zhao & L. Fan, 2019)

Investigating the Pedagogy of Mathematics: **How do teachers develop their knowledge?**

范良火◎著

教师教学知识
发展研究 第二版

华东师范大学出版社
全国百佳图书出版单位

INVESTIGATING
THE PEDAGOGY
OF MATHEMATICS

How Do Teachers Develop Their Knowledge?

LIANGHUO FAN

Imperial College Press
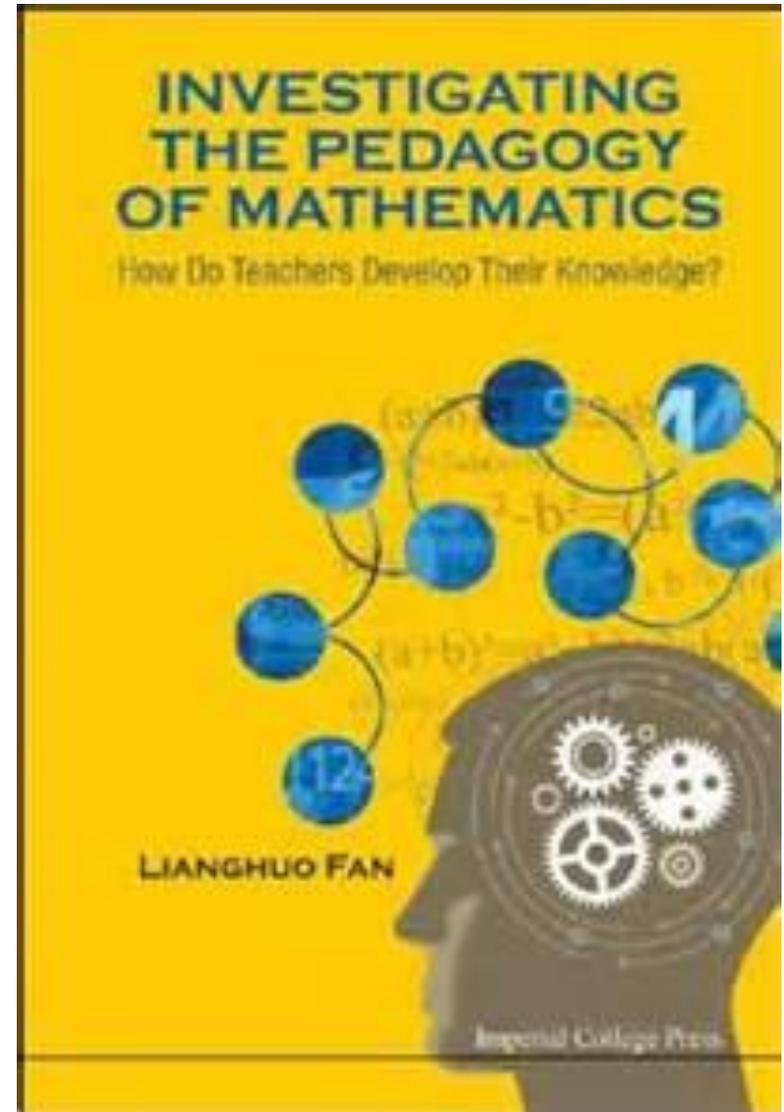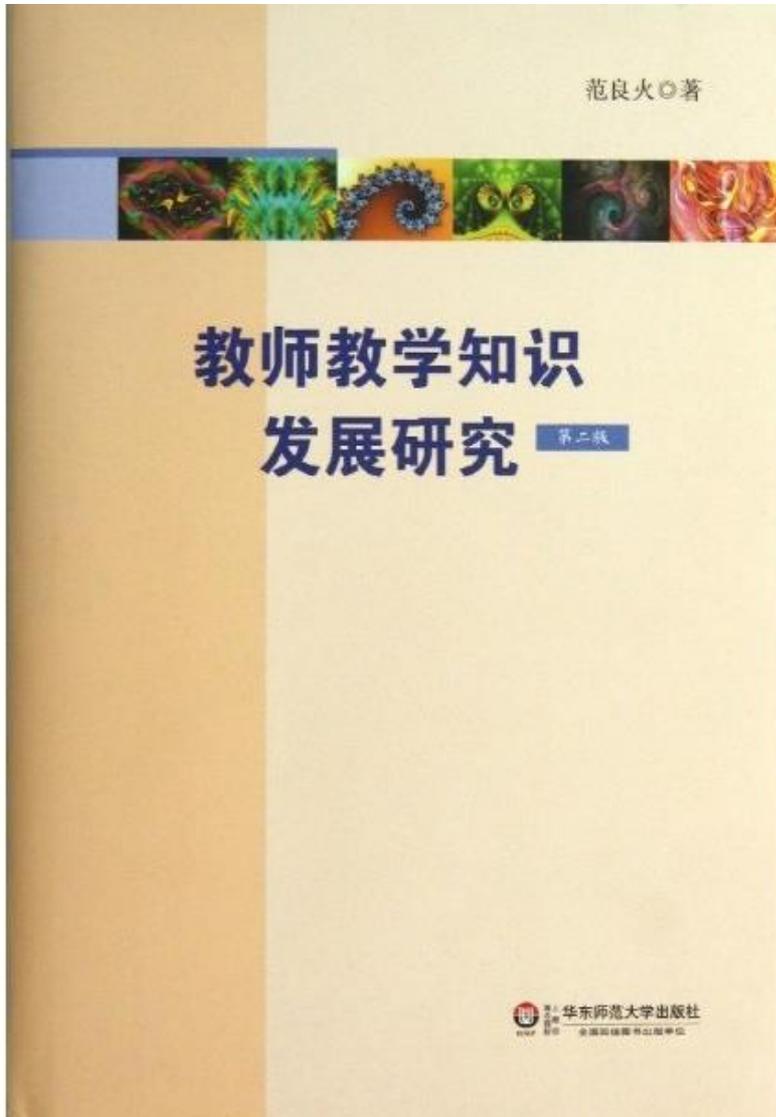
3.4 Mathematics culture and history in mathematics education

- **HPM (History and Pedagogy of Mathematics)**

- Since the First National History of Mathematics and Mathematics Education Symposium （全国数学史与数学教育研讨会）(2005), the importance of history of mathematics in mathemtics education has been gradually recognized by researchers. (Z. Shen et al., 2017)

- An example for HPM: $(a+b)(a-b) = a^2 - b^2$ could be proved by dissection and rearrangement. This method was first raised by Zhao Shuang, a Chinese mathematician in the era of Three Kingdoms. (X. Wang, 2017)
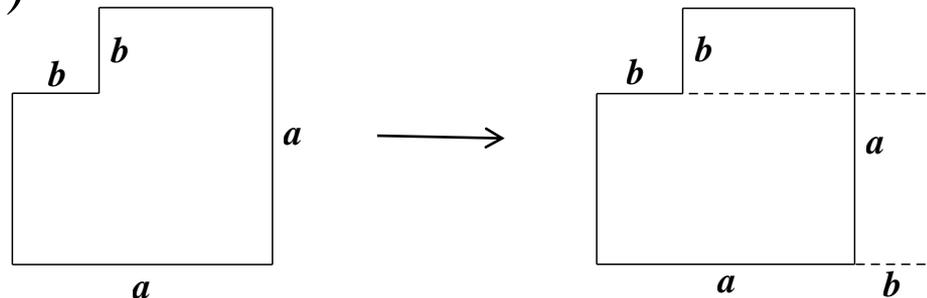

Fig. 8 The Zigzag Pattern （矩尺图）
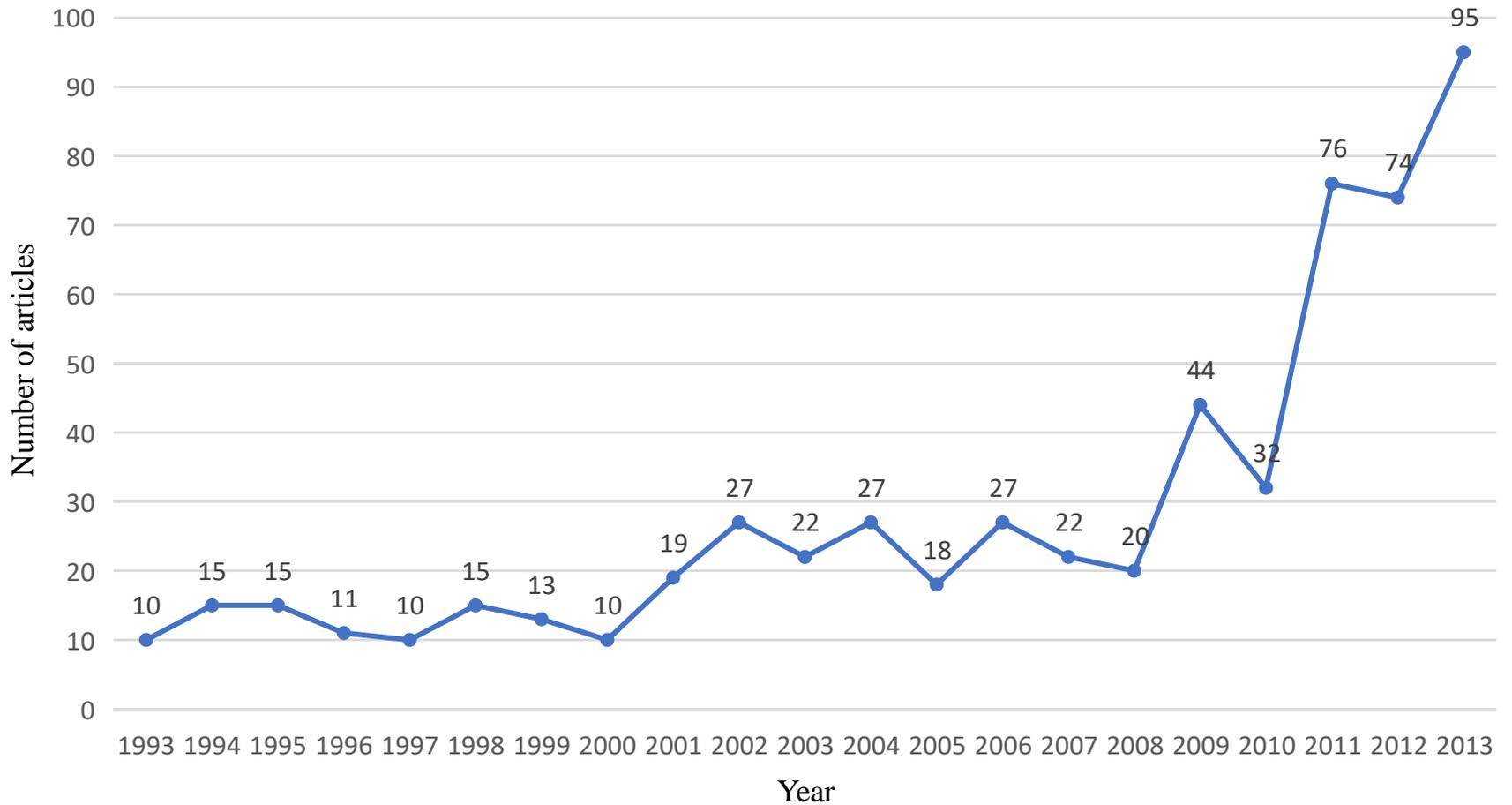
Fig. 12 Trend of articles on mathematics education of
students of ethnic minorities between 1993 and 2013 (n=602)

(D. Zhang et al., 2015)

3.5 Others

- **Issues, methods and philosophy in mathematics education**

In the 1990s, there were few articles of meta-study of mathematics education. With the development of mathematics education research, there were more articles focusing on meta-study of mathematics education.

However, there was still a lack of articles discussing the method in mathematics education research.

- **Mathematics thinking and methodology**

This issue was a quite important subject that is mostly about problem solving abilities. It is highlighted in the 1990s, but later on it lost its importance, but still it has been discussed all over the period.

4. Concluding remarks: Possible directions and trends

Possible directions and trends

1. Core Literacy

2. Mathematical modelling

3. Mathematics Curriculum and Textbook Research

4. How to develop students' higher-order thinking skills and abilities

5. Teacher Knowledge (PCK) and Professional Development

6. ICT and mathematics education

7. HPM?

Thank you

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